Embracing monotonicity in F*

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joint work with

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(based on our POPL 2018 paper)

ICE-TCS Seminar

January 29, 2018

Outline

- F*
- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

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F* [fstar-lang.org]

- **F*** is
  - a **functional programming language**
    - ML, OCaml, F#, Haskell, ...
    - extracted to OCaml or F#; subset compiled to efficient C code
  - an **interactive proof assistant**
    - Agda, Coq, Lean, Isabelle/HOL, ...
    - interactive modes for Emacs and Atom
  - a **semi-automated verifier** of imperative programs
    - Dafny, Why3, FramaC, ...
    - Z3-based SMT-automation; tactics and metaprogramming (WIP)
- **Application-driven** development
  - Project Everest [project-everest.github.io]
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F* – a prog. lang./proof assistant/verifier

```
module Talk

// Dependent (inductive) types

type vector 'a : nat -> Type =
  | Nil : vector 'a 0
  | Cons : #n:nat -> 'a -> vector 'a n -> vector 'a (n + 1)

// Dependently typed (recursive, total) functions

val append : #a:Type -> #n:nat -> #m:nat -> vector a n -> vector a m -> Tot (vector a (n + m))
let rec append #a #n #m xs ys =
  match xs with
  | Nil -> ys
  | Cons #n x xs -> Cons x (append xs ys)

// Refinement types

let in_range_index (min:nat) (max:nat) = i:nat{min <= i /\ i <= max}

val lkp : #a:Type -> #n:nat -> vector a n -> in_range_index 1 n -> Tot a
let rec lkp #a #n xs i =
  match xs with
  | Cons x xs -> if i = 1 then x else lkp xs (i - 1)

// First-class predicates (for which Type0 behaves like (classical) Prop)

type is_prefix_of (#a:Type) (#n:nat) (#m:nat) (xs:vector a n) (zs:vector a m{n <= m}) : Type0 =
  forall (i:nat) . (1 <= i /\ i <= n) ==> lkp xs i == lkp zs i

// Extrinsic reasoning (using separate lemmas)

val lemma : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> ys:vector a m -> Lemma (requires (True))
                                                                                    (ensures  (xs `is_prefix_of` (append xs ys)))
let rec lemma #a #n #m xs ys =
  match xs with
  | Nil -> ()
  | Cons x xs -> lemma xs ys

// Intrinsic reasoning (making lemmas part of definitions)

val take : #a:Type -> #n:nat -> zs:vector a n -> m:nat -> Pure (vector a m) (requires (m <= n))
                                                                            (ensures  (fun xs -> xs `is_prefix_of` zs))
let rec take #a #n zs m =
  if m > 0 then match zs with | Cons z zs -> let m' : nat = m - 1 in Cons z (take zs m')
           else Nil
```

F* – not just a pure programming language

- `Tot`, `Lemma`, `Pure`, ... are just some **effects** amongst many
  - `Tot t`
  - `Lemma (requires pre`$_{\texttt{Lemma}}$`) (ensures post`$_{\texttt{Lemma}}$`)`
  - `Pure t (requires pre`$_{\texttt{Pure}}$`) (ensures post`$_{\texttt{Pure}}$`)`
  - `Div t (requires pre`$_{\texttt{Div}}$`) (ensures post`$_{\texttt{Div}}$`)`
  - `Exc t (requires pre`$_{\texttt{Exc}}$`) (ensures post`$_{\texttt{Exc}}$`)`
  - `ST t (requires pre`$_{\texttt{ST}}$`) (ensures post`$_{\texttt{ST}}$`)`
  - ...
- **Monad morphs.** $\texttt{Pure} \leadsto \{\texttt{Div}, \texttt{Exc}, \texttt{ST}\}$; $\texttt{Exc} \leadsto \texttt{STExc}$; ...
- Systematically derived from **WP-calculi** (see POPL'17 paper)

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Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); } \texttt{assert } (\texttt{v} \in \texttt{get}())$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure}()\ \{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ **through** the proof of c_p
  - **does not guarantee** that $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ holds at every point in c_p
  - **sensitive** to proving that c_p maintains $\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s}$ for some w

- However, if c_p **never removes**, then $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get}()$ "**for free**"

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- **Programming** also relies on **monotonicity**,
  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state + general monotonicity**

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Monotonicity is really useful!

- In this talk, we will see how monotonicity gives us
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- See our POPL 2018 paper for more
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies in F*
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
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Key ideas behind our general framework

- Based on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** rel on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. rel) when
    $$\forall\, s\, e'\, s'.\ (e, s) \rightsquigarrow^* (e', s') \implies \texttt{rel}\ s\ s'$$
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- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of p s in some state s
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

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$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{ST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\,\_.\top))\ (\mathtt{ensures}\ (\lambda\,\_\,\_\,\mathtt{s}_1\,.\,\mathtt{s}_1 = \mathtt{s}))$$

- **Refs.** and **local state** are defined in F* using **monotonicity**

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

- The **get** action is typed as in ST

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- To ensure **monotonicity**, the **put** action gets a precondition

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- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\mathrm{def}}{=} s_0{:}\mathtt{state} \to \mathtt{t} * s_1{:}\mathtt{state}\{\mathtt{rel}\ s_0\ s_1\}$$

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New: Recalling a Witness

- We extend F* with a **logical capability**

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}) \to \texttt{Type}$$

  together with a **weakening principle (functoriality)**

$$\texttt{wk} : p{,}q{:}(\texttt{state} \to \texttt{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \texttt{s}\,.\, \texttt{p s} \implies \texttt{q s}))$$
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- Intuitively, think of it as a **necessity modality**

$$[\![\texttt{witnessed p}]\!](\texttt{s}) \stackrel{\text{def}}{=} \forall\, \texttt{s}'\,.\, \texttt{rel s s}' \implies [\![\texttt{p s}']\!](\texttt{s})$$

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- … Hoare-style logics are essentially **world/state-indexed**, so

- we include a **stateful introduction rule** for `witnessed`

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Outline

- F*
- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathtt{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s})\texttt{; c\_p(); recall } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s})\texttt{; assert } (\mathtt{v} \in \mathtt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\mathtt{w} \in \mathtt{get}())$$

    around the program is handled **similarly easily** by

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$$\texttt{create 0; incr(); witness } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)\texttt{; c\_p(); recall } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)$$

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

```
type heap =
  | H : h:(ℕ → cell) → ctr:ℕ{∀n. ctr ≤ n ⟹ h n = Unused} → heap
```

  where

```
type cell =
  | Unused : cell
  | Used : a:Type → v:a → cell
```

- Next, we define a **preorder** on heaps (**heap inclusion**)

```
let heap_inclusion (H h0 _) (H h1 _) = ∀id. match h0 id, h1 id with
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  | Unused, Used _ _ → ⊤
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- As a result, we can define new **local state effect**

$$\texttt{MLST t pre post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap},\texttt{heap\_inclusion}}\ \texttt{t pre post}$$

- Next, we define the type of **references** using monotonicity

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abstract type ref a = id:ℕ{witnessed (λ h . contains h id a)}
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where

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let contains (H h _) id a =
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- Important: `contains` is **stable** wrt. `heap_inclusion`

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- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read (r:ref a) : MLST a (req. (⊤)) (ens. (...)) = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write (r:ref a) (v:a) : MLST unit ... = ...`
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    - **update** the heap with the given value at the given ref.
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ML-style typed references (local state)

- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

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    $$| \ \texttt{Used} : \mathrm{a{:}Type} \to \mathrm{v{:}a} \to \mathrm{t{:}tag} \to \texttt{cell}$$

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  - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)

- See our POPL 2018 paper for
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  - **meta-theory** and **total correctness** for `MST`
    - based on an instrumented operational semantics

$$(\texttt{witness}\ x.\varphi\ ,\ s\ ,\ W)\ \rightsquigarrow\ (\texttt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

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Thank you for your attention!

Questions?

Appendix: Mon. reification and reflection

- In F* every **abstract** ST **computation**

  $$\mathtt{e} : \mathtt{ST}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

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  and vice versa using **reflection** (see our POPL 2017 paper)

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- In our POPL 2018 paper, we support reification and reflection by
  - indexing $\texttt{MST}_{\texttt{state},\texttt{rel},\texttt{b}}$ with a **boolean flag** `b` (reifiable?), and
  - **guarding** the pre-postconditions of `witness` and `recall` with `b`

  so if `b = true` then `witness` and `recall` are **logically no-ops.**

- This **works** but leads to **duplication** of pre- and postconditions!

- Instead, ongoing work is taking (hybrid) **modal logic** seriously

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